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Transverse energy flow at HERA

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Abstract

We calculate the transverse energy flow accompanying small x deep-inelastic events and compare with recent data obtained at HERA. In the central region between the current jet and the remnants of the proton we find that BFKL leading $\ln(1/x)$ dynamics gives a distinctively large transverse energy distribution, in approximate agreement with recent data.

The structure function $F_2(x, Q^2)$ for deep-inelastic electron-proton scattering has recently been measured [1] in the small x region accessible at HERA, $x \sim 10^{-3}$. The measured values show a striking rise with decreasing x which is entirely consistent with perturbative QCD expectations based on the precocious onset of the Balitsky-Fadin-Kuraev-Lipatov (BFKL) [2] leading $\ln(1/x)$ behaviour. However, the observed small x behaviour of F_2 can equally well be mimicked by conventional dynamics based on Altarelli-Parisi (GLAP) evolution [3], where the steep behaviour is either put into the starting distributions or, alternatively, generated by Q^2 evolution from a very low scale Q_0^2.

To obtain a sensitive discriminator between BFKL and conventional GLAP dynamics we need to look into the properties of the final state. A relevant observable, which has been recently measured at HERA by the H1 collaboration [4], is the transverse energy (E_T) flow in deep-inelastic events. The deep-inelastic data hint at an excess of E_T in the forward part of the central region when compared to Monte Carlo simulations which incorporate GLAP evolution. GLAP evolution corresponds to a summation of large $\ln Q^2$ terms, which is equivalent (in a physical gauge) to the summation of ladder diagrams with strongly ordered transverse momenta (k_T) of the emitted partons along the ladder: that is $Q^2 \gg k_n^2 \gg ... \gg k_1^2$, where we have omitted the subscript T on k_{Ti}^2.

At small x it becomes necessary to resum the large $\ln(1/x)$ terms and this is accomplished via the BFKL equation. The gluon 'ladder' diagrams relevant to this equation do not have the strong-ordering of the transverse momenta that is present in the GLAP diagrams. As a result more transverse energy is expected in the central region (between the current jet and the proton remnants) than would occur from conventional GLAP dynamics. These expectations are confirmed by explicit calculations [5]. They are also hinted at by Monte Carlo simulations (which incorporate BFKL effects) of the gluon radiation accompanying heavy quark production at sufficiently small x, see Fig. 8 of ref. [6] or Fig. 9(b) of ref. [7].

In ref. [5] we were mainly concerned with formalism and in using analytic methods to gain insight into the general characteristic features of the BFKL description of the energy flow in the small x regime. For fixed α_s we derived an analytic form of the E_T flow in the central region of deep-inelastic events at small x. The E_T distribution was found to be a broad Gaussian-shaped plateau as a function of rapidity, with a height that increases with decreasing x, and/or increasing Q^2. We also performed numerical estimates of the E_T flow, which included the effects of running α_s. These, more physical, calculations qualitatively confirmed the characteristic features of the fixed-α_s treatment, but did not fully cover the central region. The BFKL gluon emissions are only one source of transverse energy. There will also be contributions arising from parton radiation from the current jet and the proton remnants, and some enhancement of the E_T flow from the subsequent hadronization. Here, in order to attempt a realistic comparison with the recent data, we extend the BFKL-based calculations of E_T to cover a larger interval of rapidity and we use a Monte Carlo to simulate the effects of radiation from the current jet and of hadronization. We choose the LEPTO Monte Carlo [8], as it gives a good description of final state observables in deep-inelastic scattering (and e^+e^- collisions) in regions insensitive to BFKL small x dynamics. The matrix element (ME) + parton showering (PS) structure of the programme has the advantage that the GLAP-based initial state radiation can be isolated and therefore, in principle, be substituted by BFKL gluon emissions.

The energy flow accompanying deep-inelastic events, in a small interval about x, Q^2, is given

by [5]

$$\frac{\partial E_T}{\partial \ln(1/x_j)} \simeq \frac{1}{F_2}\int dk_j^2\, |\mathbf{k_j}| \int \frac{d^2k_p}{\pi k_p^4}\int \frac{d^2k_\gamma}{k_\gamma^4}\left(\frac{3\alpha_s}{\pi}\frac{k_p^2 k_\gamma^2}{k_j^2}\right)\mathcal{F}_2(x/x_j, k_\gamma^2, Q^2) f(x_j, k_p^2)\delta^{(2)}(k_j - k_\gamma - k_p) \tag{1}$$

where the transverse momenta are defined in Fig.1(a). For simplicity we have omitted the longitudinal structure function and assumed that $F_2 = 2xF_1$. It is straightforward to include the small correction arising from $F_L = F_2 - 2xF_1$. The function f is the unintegrated gluon distribution of the proton in which the k_p^2 integration is unfolded. To be precise

$$x_j g(x_j, \mu^2) = \int^{\mu^2} \frac{dk_p^2}{k_p^2} f(x_j, k_p^2, \mu^2) \tag{2}$$

gives the conventional gluon density at a scale μ^2. In the leading $\ln(1/x_j)$ approximation f is found to be independent of the scale μ^2 [9] and for this reason we have omitted the scale variable from the arguments of f in (1). In this small x_j approximation the function f satisfies a BFKL equation which effectively sums the soft gluon emissions below the emitted (x_j, k_j) gluon in Fig.1(a). The same remarks apply to $\mathcal{F}_2$ such that for sufficiently small x/x_j, the function $\mathcal{F}_2$ becomes scale independent and satisfies a BFKL equation which effectively sums the soft gluon emissions above the emitted gluon.

As in ref. [5], $f(x_j, k_p^2)$ is determined for $x_j < 10^{-2}$ by step-by-step integration of the BFKL equation down in x_j starting from a gluon distribution at $x_j = 10^{-2}$ obtained from the MRS set of partons of ref. [10], and $\mathcal{F}_2$ is calculated for $x/x_j < 10^{-1}$, as in ref. [11], by evolving down from the quark box (and crossed box) contribution, $\mathcal{F}_2^{(0)}$, evaluated at $x/x_j = 10^{-1}$. Even in the lowest x region accessible at HERA, $x \sim 10^{-4}$, the above QCD prediction is limited to the x_j interval $10^{-3} < x_j < 10^{-2}$. In Fig. 2(a) the continuous curve for $10^{-3} < x_j < 10^{-2}$ shows the E_T distribution for $x = 10^{-4}$ and $Q^2 = 10$ GeV^2 – values which are representative of the lowest x regime accessible for deep-inelastic scattering at HERA. The infrared cut-off on the transverse momentum integrations is taken to be $k_0^2 = 1$ GeV^2 throughout, a value for which the calculated [12] values of F_2 are consistent with the recent measurements at HERA [1]. The predictions for the E_T flow are less sensitive to the choice of the cut-off than those for F_2. The ultraviolet cut-off is chosen to be Q^2/z as implied by energy-momentum conservation [13], where $z = x/x_j$.

To extend the prediction of the E_T flow into the region $x_j > 10^{-2}$ we proceed as follows. First we continue to use formula (1), but with

$$f(x_j, k_p^2) = \left.\frac{\partial(x_j g(x_j, \mu^2))}{\partial \ln \mu^2}\right|_{\mu^2 = k_p^2} \tag{3}$$

with the gluon taken from ref. [10]. This expression for f follows from (2), since, in the leading $\ln(1/x_j)$ limit, f is independent of μ^2. However, as x_j increases we soon reach the stage when the scale dependence of $f(x_j, k_p^2, \mu^2)$ can no longer be neglected and so (3) becomes invalid. The gluon distribution f should always be positive, whereas the logarithmic derivative of xg becomes negative with increasing x_j due to the increasing importance of the usual virtual corrections of the GLAP equation. For this reason (as well as the omission of a quark contribution) the

E_T flow calculated from the above prescription will be a larger and larger underestimate as x_j increases above $x_j \simeq 10^{-2}$. This effect can be clearly seen in Fig. 2.

A better approach for these larger values of x_j is to use the usual strong ordering at the gluon emission vertex, $k_j^2 \gg k_p^2$, so that $k_\gamma^2 \approx k_j^2$. Then, on making use of (2), eq.(1) simplifies to

$$\frac{\partial E_T}{\partial \ln(1/x_j)} = \frac{1}{F_2} \int \frac{dk_j^2}{k_j^4} \frac{3\alpha_s}{\pi} |\mathbf{k_j}|\, x_j \sum_a f_a(x_j, k_j^2) \mathcal{F}_2(x/x_j, k_j^2, Q^2), \tag{4}$$

where the "effective" parton combination $\sum_a f_a \equiv g + \frac{4}{9}(q + \bar{q})$ arises from the dominance of gluon exchange [14]. In this way we include the contributions when parton a of Fig.1(b) is either a quark or an antiquark, as well as the gluonic component which was dominant in the smaller x_j region. For sufficiently large x_j (but away from the proton remnants) formula (4) will give a much more reliable prediction for the transverse energy flow, but as x_j decreases the strong-ordering assumption becomes less valid and the consequent neglect of regions of phase space means that the E_T flow will again be underestimated. This is apparent from Fig. 2(a) which shows the predictions of (4) as a continuous curve in the interval $0.01 < x_j < 1$. In Fig. 2(b) we show the results for $x = 5.7 \times 10^{-4}$ and $Q^2 = 15$ GeV^2. This choice represents the average values [15] of the variables for the E_T distribution, accompanying the deep-inelastic events with $x < 10^{-3}$, which was observed by the H1 collaboration [4]. Again there is a reasonably flat plateau in the central region, but about 0.4 GeV lower than that of Fig. 2(a) and covering a smaller rapidity interval on account of the larger value of x. In summary, in the region $10^{-2} \lesssim x_j \lesssim 10^{-1}$ the E_T flow is underestimated by both (1) and (4), but at different ends of the interval. Formula (1) is valid for $x_j \lesssim 10^{-2}$, while formula (4) is reliable for $x_j \gtrsim 10^{-1}$. From the combination of the results shown in Fig. 2 we conclude that BFKL radiation (which accompanies deep-inelastic events) gives rise to an approximately flat E_T distribution in the central region with a height, which increases slowly with decreasing x, of about 2 GeV per unit of rapidity, in the HERA small x regime.

We also performed the above calculations using GLAP evolution along the ladders. Figure 2 shows that, as expected, this evolution gives a much smaller transverse energy flow than BFKL evolution. The discontinuity in the GLAP results at $x_j = 10^{-2}$ is due to the inclusion of the quark distributions in the calculation for the large x_j region. As expected the quarks have little importance at small x_j where the gluon dominates.

It is useful to translate the energy flow $\partial E_T/\partial \ln x_j$ into a distribution in terms of rapidity, y in the HERA frame. First we note that in the virtual photon-proton centre-of-mass (cm) frame the rapidity is given by

$$y(\text{cm}) = \frac{1}{2} \ln\left(\frac{x_j^2 Q^2}{x k_j^2}\right). \tag{5}$$

In regions where the distribution is reasonably flat, a good estimate of the y distribution is obtained if we insert the local average value of k_j^2 into (5). Secondly we translate from the virtual photon-proton cm frame to the HERA laboratory frame using the formula

$$y - y(\text{cm}) \simeq \frac{1}{2} \ln\left(\frac{4xE_p^2}{Q^2}\right) \tag{6}$$

which is valid at small x, since then the frames are approximately collinear. E_p is the energy of the incoming proton in the HERA frame.

At this stage our numerical results, shown in Fig. 2(b), cannot be compared directly with the H1 data [4]. The distributions of Fig.2 correspond to E_T arising from gluons radiated from the initial state. Admittedly, at small x, this is expected to be the dominant effect in the central region between the current jet and the proton remnants – the large energy flow that is predicted is characteristic of BFKL dynamics. However the calculation does not take into account the current jet and its associated radiation, nor does it include any effects of hadronization. We therefore need to estimate the importance of the various components contributing to the transverse energy flow. To this end we compare in Fig.3 the H1 measurement of the E_T flow [4] with four different distributions obtained using the LEPTO Monte Carlo [8]. The parameters of LEPTO have been tuned to the hadronization resulting from jet production in e^+e^- collisions and, to some extent, tuned to EMC deep-inelastic data in the higher x regime [16]. The four E_T distributions resulting from LEPTO show the energy flow obtained (i) from only parton showers radiated from the current jet, (ii) with parton showers from the initial state incorporated, (iii) from only parton showers from the current jet but with hadronization effects included, and (iv) from parton showers from both the current jet and the initial state together with hadronization. We see that the Monte Carlo gives a good description of the current jet and its associated radiation, but that effects of initial state radiation and of hadronization are unable to give sufficient E_T in the forward part of the central region at small x. This has prompted a further study of the LEPTO Monte Carlo in order to assess whether this deficiency at small x is genuine or if it can be tuned away [16]. This requires looking in detail at the LEPTO modelling of the GLAP-based initial state parton showers, and of hadronization of proton remnants more complicated than those arising from a diquark. We note from ref. [4] that a Monte Carlo based on the colour dipole model appears to give a better description of the E_T flow in the central region than does LEPTO. On the other hand the colour dipole model is less successful than LEPTO in describing energy-energy correlations [4] and the Q^2 dependence of the jet rates [17]. It remains to be seen how much is simply parameter tuning and how much is directly attributable to QCD dynamics in the various Monte Carlo simulations.

In Fig. 4 we confront the BFKL predictions with the HERA measurements of the E_T flow in the central region with $x < 10^{-3}$. The average values of the deep-inelastic variables for these data correspond to $x = 5.7 \times 10^{-4}$ and $Q^2 = 15$ GeV^2 [15]. The BFKL predictions are thus simply those of Fig. 2(b), but shown now in terms of y, the rapidity in the HERA frame. This translation is achieved via eqs (5) and (6). However, the comparison of the observed E_T flow at small x with the BFKL-based estimates is clearly incomplete. As emphasized above, we have omitted the effects of hadronization and of radiation from the current jet. The magnitude of these effects can be estimated from the LEPTO Monte Carlo. The appropriate histogram of Fig. 3 is reproduced in Fig. 4.

To obtain a first estimate of the total E_T flow we could simply add the E_T resulting from the BFKL emissions to the LEPTO distribution. In other words the hadronization effects, which in LEPTO arise from the colour string stretching from the current quark to the diquark proton remnants, are assumed to give an underlying rapidity plateau whose gross features are insensitive to the properties of the initial state radiation. The Monte Carlo results in Fig. 3 give support for this assumption, since we see that the level of hadronization is approximately independent of whether or not we include parton showers from the initial state.

If such a straightforward addition were to be performed on the results shown in Fig. 4 then the total estimate of the E_T flow would be in approximate agreement with the data. There are two caveats. First, there may be some danger of double counting of radiation from the current jet and, second, our simple additive treatment of hadronization may be too naïve. However, the first problem does not effect the central region, and secondly hadronization effects appear to be much less than the BFKL signal.

To conclude, we have shown that small x deep-inelastic scattering is accompanied by a large E_T flow in the central region arising from soft gluon radiation. This is a hallmark of BFKL dynamics and arises from the relaxation of the strong-ordering of transverse momenta. The first experimental measurements of the E_T flow in small x deep-inelastic events indicate that there is significantly more E_T than is given by conventional QCD cascade models based on Altarelli-Parisi evolution. Instead we find that they are in much better agreement with estimates which incorporate BFKL evolution. The latter dynamics are characterised by an increase of the E_T flow in the central region with decreasing x. Measurements of the energy flow in different intervals of x, in the small x regime, should therefore allow a definitive check of the applicability of BFKL dynamics.

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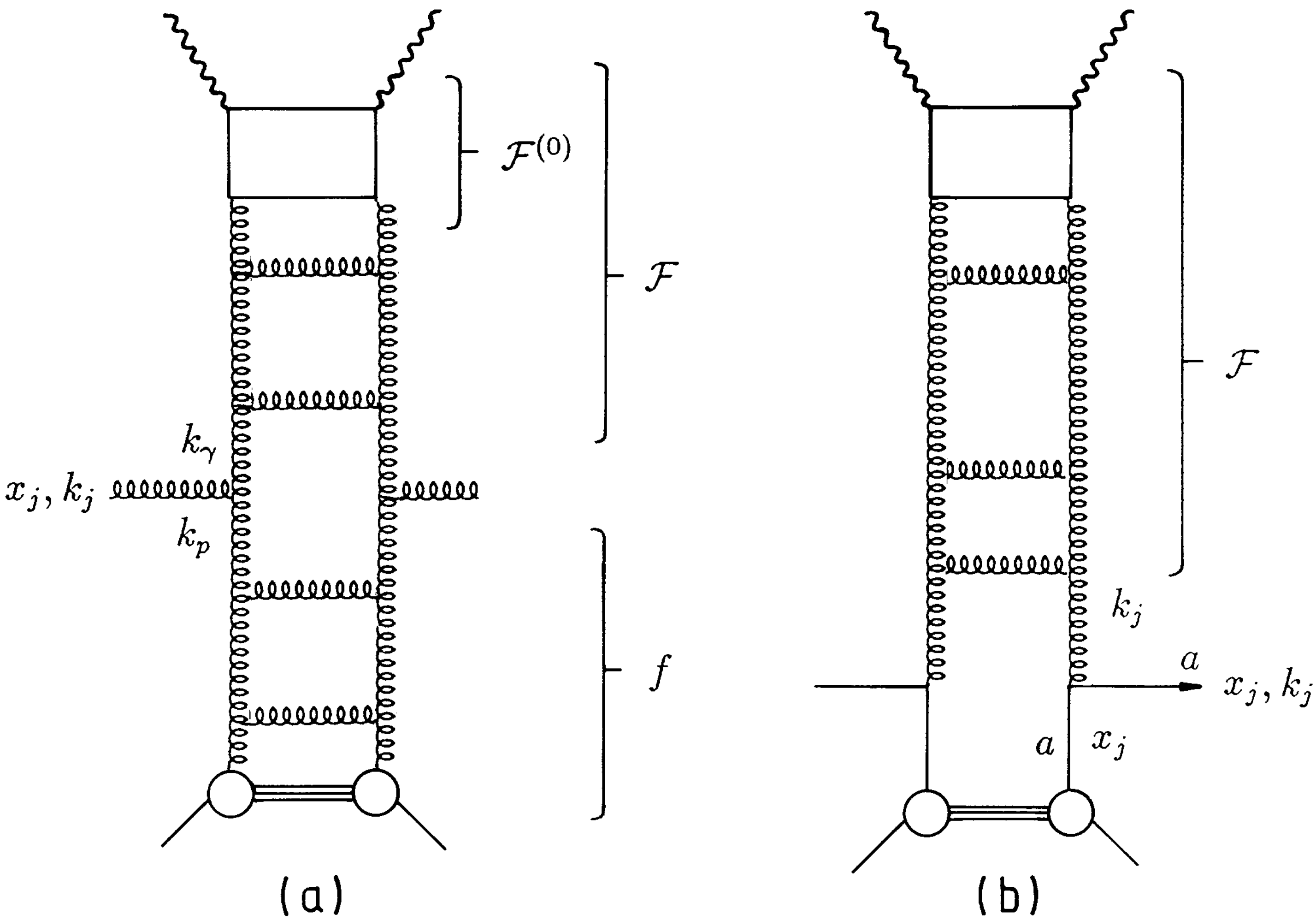
Figure Captions

Fig. 1: (a) Diagrammatic representation of formula (1) showing the gluon ladders which are resummed by the BFKL equations for f and $\mathcal{F}_2$. (b) The representation of formula (4), which is obtained by the simplification of (1) when x_j is large; there is now strong-ordering at the (parton a)–gluon vertex.

Fig. 2: Numerical calculation of the E_T flow as a function of x_j for (a) $x = 10^{-4}, Q^2 = 10$ GeV^2 and (b) $x = 5.7 \times 10^{-4}, Q^2 = 15$ GeV^2. The latter choice of variables is relevant to the HERA data for $x < 10^{-3}$. The effects of the current jet (and its associated radiation) and of hadronization are not included. The continuous curves are based on BFKL dynamics: formula (1) is used for $x_j < 10^{-2}$ and formula (4) is used for $x_j > 10^{-2}$. For comparison, the dashed curves show the E_T flow calculated using GLAP evolution. The dotted curve shows the effect of including only gluons at large x_j in the GLAP evolution.

Fig. 3: The data show the E_T flow as a function of rapidity in the laboratory (HERA) frame which accompanies deep-inelastic events with $x < 10^{-3}$ [4]. The proton direction is to the right. The LEPTO Monte Carlo distributions correspond, in increasing order, to ME+PS(final), ME+PS(initial,final), ME+PS(final) + hadronization, and finally ME+PS(initial,final)+ hadronization.

Fig. 4: The data show the E_T flow accompanying deep-inelastic events with $x < 10^{-3}$ observed by the H1 collaboration [4] in the central region. The continuous curves show the BFKL predictions of $x = 5.7 \times 10^{-4}$ and $Q^2 = 15$ GeV^2, which correspond to the average values of the variables for the data sample. The histogram is the LEPTO Monte Carlo estimate from Fig. 3 of the effects of radiation from the current jet and of hadronization.



$\mathcal{F}^{(0)}$
$\mathcal{F}$
k_γ
x_j, k_j
k_p
f
(a)
$\mathcal{F}$
k_j
a
x_j, k_j
a
x_j
(b)

Fig. 1

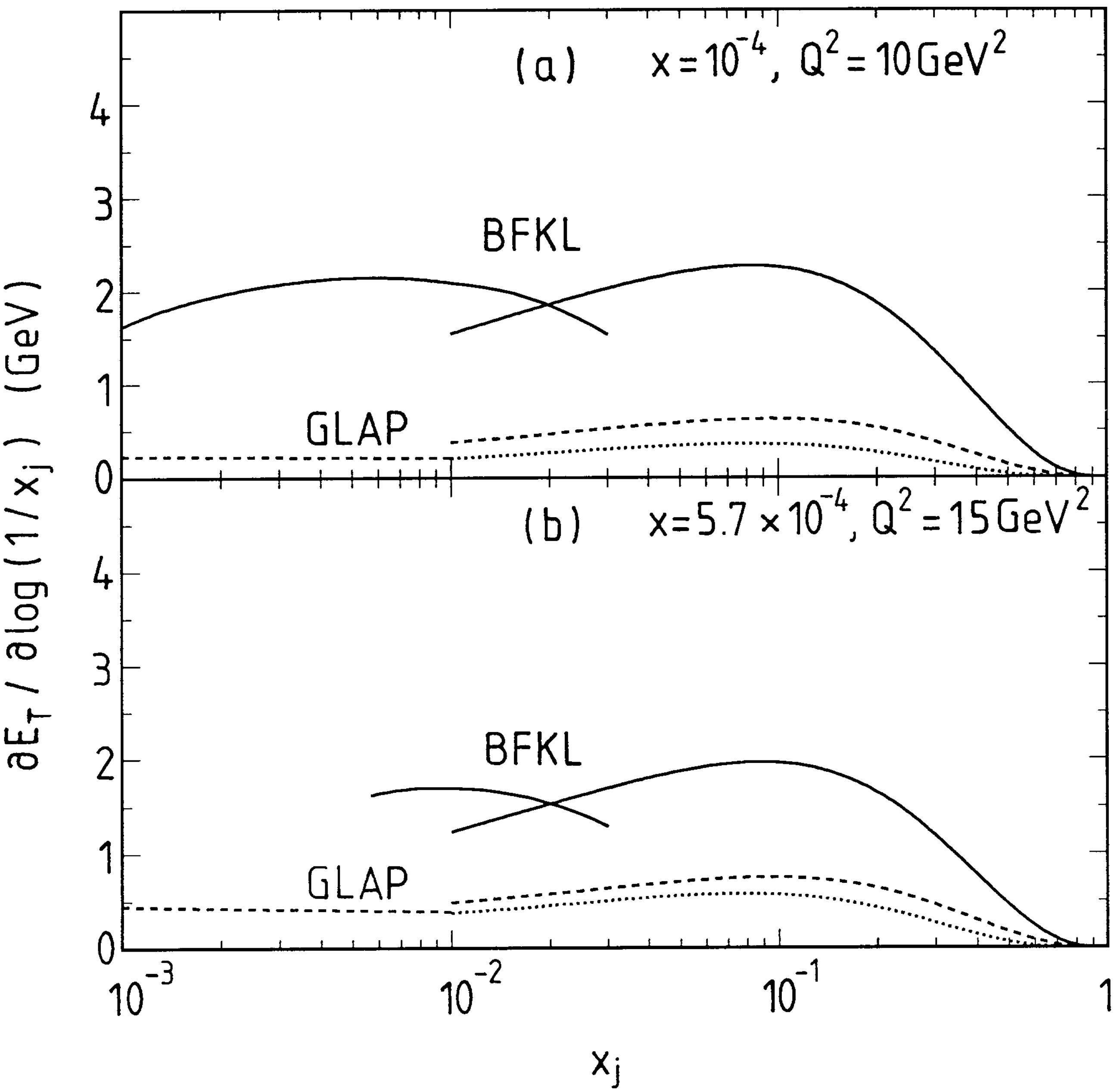
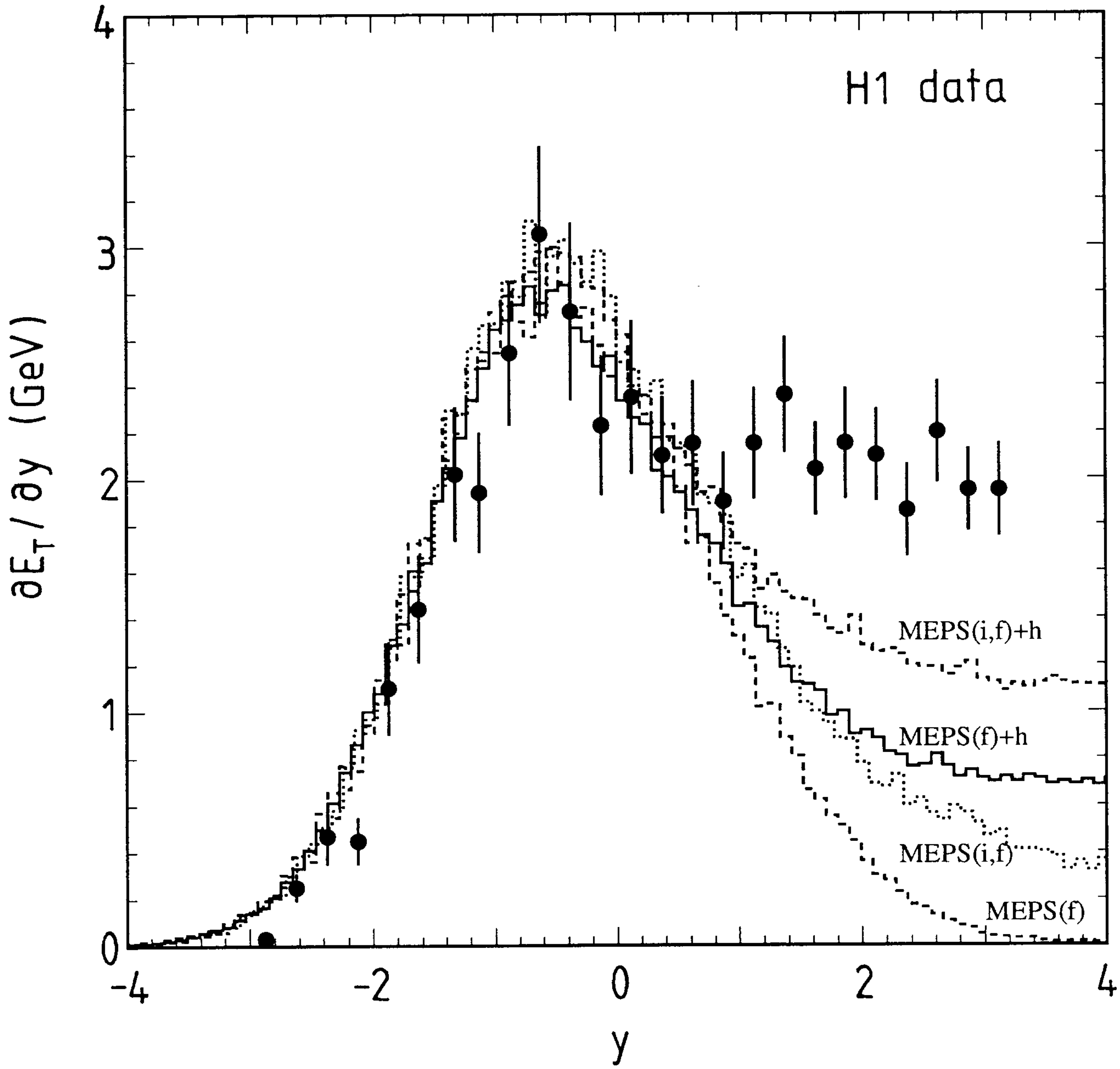

(a) $x = 10^{-4}$, $Q^2 = 10\,\mathrm{GeV}^2$
(b) $x = 5.7 \times 10^{-4}$, $Q^2 = 15\,\mathrm{GeV}^2$
BFKL
GLAP
BFKL
GLAP
$\partial E_T / \partial \log(1/x_j)$ (GeV)
x_j
0
1
2
3
4
10^{-3}
10^{-2}
10^{-1}
1

Fig. 2



H1 data
MEPS(i,f)+h
MEPS(f)+h
MEPS(i,f)
MEPS(f)
$\partial E_T / \partial y$ (GeV)
y

Fig. 3

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http://arXiv.org/ps/hep-ph/9405400v1

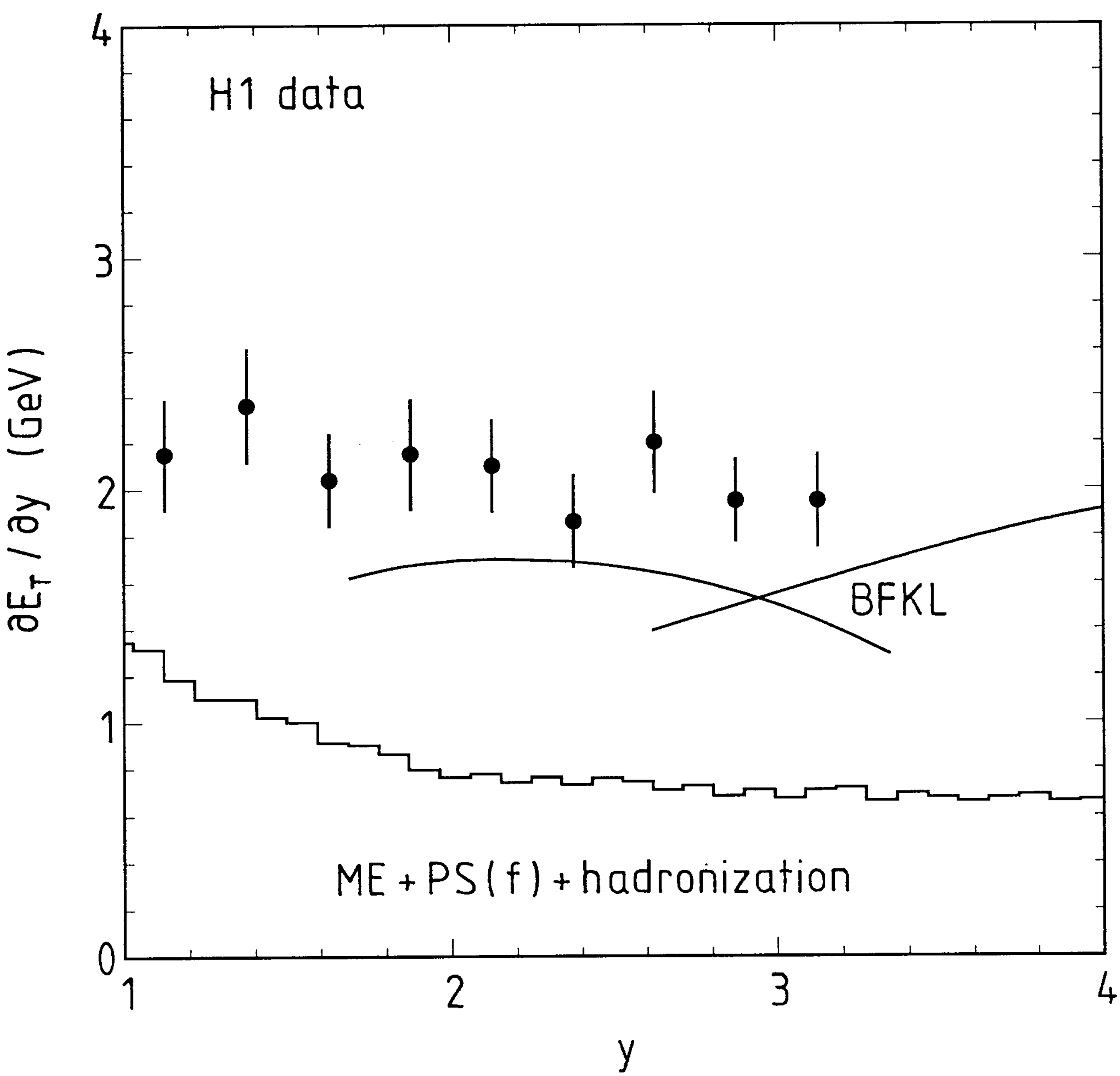

Fig. 4